Sudden Transitions in Attitudes

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Both the dynamic approach and catastrophe modeling have been warmly welcomed in research on attitudes and opinions. In this article, the authors discuss a general methodology for testing catastrophe models and apply it to the dynamics of attitude formation and change. First, by making use of the so-called catastrophe flags, converging support for the catastrophe model can be attained. Each flag relates to a specific hypothesis about attitudinal change. Second, fitting stochastic catastrophe models to data enables one to carry out a direct test of catastrophe models. Results of analyzing large data sets on political attitudes support the validity of the general catastrophe model of attitude change in which transitions in attitudes are a function of involvement and information. Present results suggest that in the case of political attitudes, involvement might well be correlated with attitude. A more refined approach to the measurement of information and involvement is suggested.

Keywords: *attitude change; polarization; ambivalence; stochastic catastrophe theory; catastrophe flags; maximum likelihood estimation*

INTRODUCTION

The dynamical system approach is the subject of increasing interest in psychology in general. More recently, the approach has been applied to attitudes, and several authors have stressed the potential of the dynamical system approach. Petty, Wegener, and Fabrigar (1997) consider the dynamical system approach to the study of attitudes an important new research area. McGarty and Haslam (1997) refer to the dynamical system approach as one of the most promising new developments in social psychology. Catastrophe theory (Thom 1972;

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Zeeman 1976; Gilmore 1981), part of the dynamical system approach, has been applied to attitudes to explain and describe transitions in attitudes (see, e.g., Zeeman 1976; Flay 1978; Latané and Nowak 1994; Tesser and Achee 1994).

According to these authors, attitudes are unstable when involvement is high and arguments, pro and contra an attitude, balance each other out. In such a transitional state, small changes in pro/contra information may lead to large, sudden jumps in attitudes. Although the application of catastrophe theory holds promise, several researchers stress the need for sound statistical methods that can be applied to catastrophe hypotheses and dynamical system hypotheses in general (Beek, Verschoor, and Kelso 1997; Burlingame and Hope 1997; Kruglanski, Clement, and Jost 1997).

Several early investigators (Zeeman 1976; Flay 1978; Tesser 1980) proposed appealing catastrophe models but never tested them directly. This was mainly because standard statistical tests cannot simply be applied to catastrophe models. These early investigators were heavily criticized (Sussman and Zahler 1978), mainly for the lack of testing. In response to this criticism, several researchers developed methods to test catastrophe models.

A first line of research involves the testing of specific predictions derived from catastrophe models (the so-called catastrophe flags) (Gilmore 1981). The more that these catastrophe flags are detected, the more convincing is the claim of a catastrophe. As will be shown, this line of research mainly consists of the integration and reinterpretation of empirical results and ideas put forward in the attitude literature. A second line of research focuses on techniques that apply explicit statistical formulations of the catastrophe models. These techniques involve the direct statistical fit of the catastrophe model to empirical data (Cobb 1978, 1980, 1981; Guastello 1988; Oliva, Desarbo, Day, and Jedidi 1987). As explained below, there are good reasons to prefer the method of Cobb. Computational improvements make his method very useful (Hartelman 1997; Hartelman, van der Maas, and Molenaar 1998).

We discuss and apply both lines of testing within the catastrophe model for attitude change to cross-sectional data. With these applications, we intend to test a model of attitude change and demonstrate that testing catastrophe models in psychology is feasible.

THE CATASTROPHE THEORY OF ATTITUDES

Zeeman (1976) introduced catastrophe theory in psychology and also formulated the specific hypothesis that (attitude) change can be described by catastrophe theory (Thom 1972; Poston and Stewart 1978; Gilmore 1981; Castrigiano and Hayes 1993). This hypothesis implies that smooth changes in independent or control variables may lead to abrupt, discontinuous changes in attitudes. To understand the details of Zeeman's hypothesis, some understanding of catastrophe theory is required.

There are seven different families of catastrophe models, based on the number of control and dependent variables. In the social sciences, the so-called cusp catastrophe model is the most frequently used model because it is the simplest catastrophe model that gives rise to sudden discontinuities. It consists of two control variables and one behavioral variable. Control variables change at least an order of magnitude slower than the behavioral variable. The cusp model is based on the following nonlinear deterministic dynamical system:

$$dX/dt = -dV(X;\alpha,\beta)/dX.$$

That is, behavior X (attitude in our case) changes over time t according to the derivative of the cusp potential function:

$$V(X;\alpha,\beta) = \frac{1}{4}X^4 - \frac{1}{2}\beta X^2 - \alpha X,$$

which has as equilibria (first derivative to zero)

$$X^3 - \beta X - \alpha = 0,$$

where α and β are the control variables. When the values of the control variables remain fixed, the system seeks an equilibrium state (called a point attractor). This means that some minimum or maximum of a certain quantity (e.g., energy, profit, or cognitive dissonance) is obtained. Catastrophe theory explains how these equilibria change as a function of the control variables. This change of equilibria may lead to discontinuous, abrupt changes in behavior. The two control variables of the cusp are called the normal (α) and the splitting factor (β).

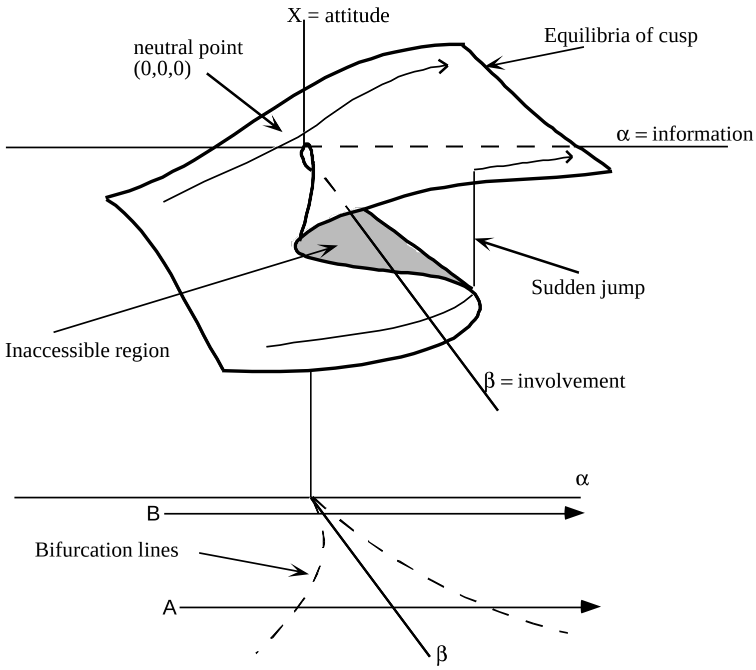

Figure 1: The Cusp Model of Attitude Change

Figure 1 gives a graphical representation of the cusp model. The folded surface represents the equilibria of the system. The plane below is the control plane. The surface between the bifurcation lines is called the bifurcation set. Outside this set, only one stable state exists. Within the set exist three states, but the middle state is repelling (i.e., a state that is inaccessible). Sudden catastrophic jumps occur when the normal variable is smoothly changed at high values of the splitting variable. A typical example is the freezing of water. The condition of water is the dependent variable, and the two independent variables are temperature (≈ normal variable) and pressure (≈ splitting variable). Smooth changes in temperature lead to sudden transitions between the two possible states (Path A). If pressure is low (Path B), a continuous change between water and ice is possible.

The catastrophe theory of attitudes (Zeeman 1976; Flay 1978; Latané and Nowak 1994) states that information and involvement can serve as the control factors (α and β) in the cusp model. The behavioral aspect of an attitude is the equilibrium state based on information and involvement. Latané and Nowak (1994) state that this holds for stereotypes as well (see also Eiser 1994). The information variable can be perceived as a position on a dimension, which combines factors such as previous experience, self-interest, genetic disposition, and environmental effects (Zeeman 1976). Involvement is an organizing property that may be voluntary or involuntary (Zeeman 1976). Latané and Nowak state that, because attitudes represent some kind of organized body of knowledge (such as a schema or category), increasing organization should lead to attitudes becoming more categorical. Involvement features as the splitting factor because it breaks the unimodal structure of the attitude distribution into two (opposing) attitudinal positions. Involvement can also be related to the literature on attitude strength. For instance, Krosnick and Petty (1995) argue that involvement is one of the determinants of attitude importance and attitude strength.

Latané and Nowak (1994) attempted to test the cusp catastrophe for attitude change. In a first study, they reanalyzed data (from Stouffer et al. 1950) consisting of the opinions of 75,000 soldiers about 7,000 attitude statements. Latané and Nowak found that as involvement increased, bimodality in the frequency became more extreme. To test this statistically, they focused on attitude variance, assuming that increased variance would provide some evidence of multimodality and divergence. However, whereas bimodality almost always implies increased variance, increased variance does not necessarily imply bimodality. As a consequence, their statistical test is not completely convincing. In their second study, Latané and Nowak asked 100 undergraduate psychology students to judge a list of political statements in terms of favorableness and importance. They found that as judged importance increases, so does favorableness. Their results showed a significant overall (positive) correlation between importance and extremity of opinion. A replication of this study showed similar results (Latané and Nowak 1994). This correlational test is also circuitous. Although encouraging, these studies thus provide only modest support for the catastrophe model. By reanalyzing these data with the cusp-fitting technique, we will provide a more direct test of this model.

Gilmore (1981) derived eight behavioral properties of the cusp model, the so-called catastrophe flags. These catastrophe flags are (multi) modality, sudden jump, inaccessibility, hysteresis, divergence, anomalous variance, divergence of linear response, and critical slowing down. In the next section, we review each of the flags with respect to attitudinal change.

HYPOTHESIS TESTING BY USING THE CATASTROPHE FLAGS

Catastrophe theory provides a parsimonious description of many phenomena. By making use of catastrophe flags, many theories can be accommodated in a single model (van der Maas and Molenaar 1992). Below, these flags are related to specific hypotheses within the theory of attitude formation and change.

Multimodality of behavior means that for specific values of the control variables, a multimodal distribution of the behavioral variable is likely to occur. Usually, multimodality is studied in cross-sectional samples, but it can also be evaluated with individual repeated measurements (sudden jump flag). Multimodality of attitudes refers to a multimodal distribution of attitude-related behavior within a given group. Flay (1978) gives several examples of bimodality in which involvement is the splitting factor. One example concerns student evaluations of courses. According to Flay, overall ratings will be extreme (positive or negative) when workload (involvement) is rated as high. One way to assess the modality of attitudes in a statistically sound way is to use finite mixture models (Titterington, Smith, and Makov 1985). It is, for instance, possible to typify the behavior by using latent class analysis (McCutcheon 1987; van der Maas 1998).

Inaccessibility is strongly related to modality. It implies that intermediate (behavioral) states between two opposite attitudes are rare. People will behave according to one or the other attitude but will not readily assume intermediate positions. In Flay's (1978) example, students show less neutral attitudes when workload is high. The relation between involvement and attitude extremity is also addressed in the classic work of Sherif, Sherif, and Nebergall (1965).

A *sudden jump* (Path A, Figure 1) takes place when the amount of information in favor of the opposite attitude is increased continuously. Beyond some threshold, people suddenly switch from one attitude to the other. This relates to conversion in the conversion model of stereotype change (Rothbart 1981). However, instead of an attitude change after a dramatic counterattitudinal piece of information (as proposed by the conversion model), a sudden jump can occur given a very minor piece of information or following a continuous flow of counterattitudinal information. There is some anecdotal support for the hypothesis concerning the sudden transition of attitudes (e.g., the case of Patty Hearst) (see Zimbardo, Ebbesen, and Maslach 1977). When involvement is relatively low, attitude change is continuous. This resembles the bookkeeping model of stereotype change (Rothbart 1981). This model states that any gradual incoming information is weighted and added, which results in continuous change of the stereotype. Tourangeau (1987) and Saris (1997) showed that public opinion, on issues of low interest, can easily be swayed by specific contextual information.

Hysteresis means that a sudden jump will occur at different values of the normal variable, depending on the direction of change in this variable (Path A). For instance, in disturbance-free conditions, water freezes at –4 °C and thaws at 0 °C. Hysteresis in the attitude model means that the informational value at which people change their attitude (the threshold) depends on the person's initial position and the direction of change in information. For instance, people with a positive attitude toward abortion change to a negative attitude only when information is strongly against abortion. People with a negative attitude change to a positive attitude only when information is strongly in favor of abortion. Reardon and Tesser (1982) provide some experimental support of hysteresis. Respondents with a history of conformity (i.e., "weak" feelings about prior information) jump to nonconformity only if information elicits strong feelings of disagreement. On the other hand, respondents with a history of nonconformity (i.e., strong feelings about prior information) jump to conformity only if they are confronted with counterattitudinal information with which they moderately agree. The catastrophe model is unique in that it provides an explanation for the phenomenon of hysteresis. Hence, we consider this flag to be sufficient to conclude that a catastrophe model holds.

When the splitting factor increases, the multimodality of the behavior becomes more extreme. This is called *divergence* because two initially proximate behaviors become increasingly opposed to each other. Divergence in the attitude model means that two (groups of) persons with initially almost the same neutral attitude (an attitude based on an intermediate value of information) can diverge into two opposites when involvement increases. This corresponds to the notion of polarization (Latané and Nowak 1994; Latané, Nowak, and Liu 1994; Tesser 1976). Attitudes that differ slightly may diverge into strong opposite attitudes. An example of divergence can be found in an experiment by Teahan (1975), who found a polarization effect among Black and White police officers. At their entrance into the police academy, attitudes of these officers were similar. After an 18-month period, both groups showed more interest in their own group and more hostility toward the other group.

Anomalous variance is an increase in the variance of behavior that occurs in the neighborhood of the bifurcation set (e.g., in the area where sudden transitions are possible). In this area, we can expect large fluctuations in behavior. For instance, people who have just arrived in some new group (e.g., immigrants who have settled in a new country) have to come to understand the attitudes of the new group and to give meaning to the behaviors that surround them. They will fluctuate between "old" and "new" behaviors until they settle down in the new stable behavioral state. This relates to findings concerning attitudinal ambivalence. Ambivalence occurs when people consider attitudinal information both pro and contra a given subject. Ambivalent attitudes are attitudes that simultaneously contain positive and negative cognitions (see Eagly and Chaiken 1993; Kaplan 1972; Thompson, Zanna, and Griffin 1995).

The following flags differ from those discussed above because they are associated with perturbations of the system. *Divergence of linear response* means that, after a strong counterattitudinal message, people in the neighborhood of the bifurcation area show large fluctuations in behavior before returning to their stable former attitudinal state or settling in the new one. This corresponds to the conversion model (Rothbart 1981) in which a single counterattitudinal message may result in attitude change. After some oscillations, the respondent may resort to his or her original viewpoint but may also jump to a new attitude. Depending on the complexity of the issue, subtyping (i.e., creation of new subgroups) is also possible (Weber and Crocker 1983). The counterattitudinal message is evaluated as a subcategory, which leaves the overall stereotype intact but perhaps a little less stable. Catastrophe theory predicts that when a sufficient number of counterattitudinal messages are given, even subtyping will eventually cease, and a jump to the other attitude will occur. This corresponds to the view of Pettigrew (1981), although other researchers (e.g., Taylor 1981) believe that subtyping will lead only to new, more detailed stereotypes.

Critical slowing down is closely related to the former catastrophe flag. It means that, after a perturbation, it takes a certain amount of time before stable behavior returns. Reaction times to the attitudinal items can be used to test for this. In the neighborhood of the bifurcation area, reaction times are much slower than outside the bifurcation area (van der Maas, Raijmakers, Hartelman, and Molenaar 1999). In the case of ambivalence, reaction times when judging the attitudinal object or issue are also found to be much slower (van der Pligt, De Vries, Manstead, and van Harreveld 2000). Vallacher, Nowak, and Kaufman (1994) explored mixed valence representations both conceptually and operationally. Their "mouse approach," in which they register the exact computer mouse movements in response to attitudinal information, enables one to track the time course of switching between different attitudinal positions.

FITTING CATASTROPHE MODELS

Several statistical methods to fit catastrophe models have been developed. First, we briefly discuss the main techniques.

GUASTELLO'S POLYNOMIAL REGRESSION TECHNIQUE

Guastello's (1988) polynomial regression technique takes $dX/dt = -dV/dx = -X^3 + \beta X + \alpha = 0$ as a starting point. The idea is that given some measurement of X, β, and α at $t = 1$, X at

$t = 2$ can be derived from the formula for dX/dt. Setting dt equal to 1 gives the following regression equation:

$$y_1 - y_2 = c_0 + c_1 y_1^3 + c_2 y_1^2 + c_3 \beta y_1 + c_4 \alpha,$$
$$\text{where } y_1 = (X_1 - \lambda)/\sigma \text{ and } y_2 = (X_2 - \lambda)/\sigma.$$

Guastello (1988) uses reverse hierarchical entry (entering higher order terms first) in computing the regression equation and compares the model with two alternative linear regression models. Guastello's method has been criticized by Alexander, Herbert, Deshon, and Hanges (1992) on a number of points. Their most important criticism relates to the fact that the analysis of completely random data (e.g., random numbers for y_1 and y_2) yields a proportion of explained variance of about .49 (this can be easily be demonstrated by means of simulation). This is a consequence of using the difference between y_1 and y_2 as the dependent variable and a function of y_1 as the predictor. Given some weak assumptions, if the correlation between y_1 and y_2 is 0, then the correlation between y_1 and the difference between y_1 and y_2 is about .72 (for details, see Alexander et al. 1992). In combination with the reverse hierarchical entry method, in which the cubic term is entered before the linear term, this statistical artifact leads to the incorrect conclusion that the cubic term is a significant predictor in the regression when the data consist entirely of noise.

In his reply, Guastello (1992) argues that differences between constrained random numbers are indeed catastrophically distributed. He used a fit of random numbers to the cusp model to support his view and found the above-mentioned proportion of explained variance of .49 (see fourth row of Table 2 in Guastello 1992), as predicted by Alexander et al. (1992). However, that some types of chaotic time series, which look like completely random time series, can be generated with cusp-like equations (which is what Guastello argues) does not imply that computer-generated random numbers (uniformly distributed) are catastrophically distributed.

Consider also the alternative linear models used by Guastello (1988). In both linear models, the difference between y_1 and y_2 is not a function of y_1. We think that the correct alternative linear model is Guastello's nonlinear regression equation (see above) without the quadratic and the cubic term (e.g., $c_1 = c_2 = 0$). For random data, this linear model also yields an explained variance of about .5, which illustrates two things: (a) this type of difference regression equation can be misleading,[1] and (b) the good fit of the random numbers is not related to the cubic term, and hence with the cusp model, but is an artifact of the difference method.

GEMCAT

Oliva et al. (1987) proposed a multivariate methodology for estimating catastrophe models. Lange, Oliva, and McDade (2000) improved this methodology and developed a program called Gemcat II. It is claimed that the program can fit any of the catastrophe models to data. It needs to be added that experience is mostly limited to the cusp. In Gemcat II, the cusp variables X, β, and α are all linear functions of one or more measured variables. The coefficients of these linear functions are estimated by minimizing

$$\Sigma e_t^2 = \Sigma_t [X_t^3 - \beta_t X_t - \alpha_t]^2,$$

where e_t is the error, ideally zero, for each case or data point.

This technique does not rely on difference scores and, consequently, does not suffer from the main problem of Guastello's (1988) approach. However, despite its advantages above Guastello's technique, there are two major problems with Gemcat. The first problem is that e_t is not the quantity that should be minimized in case of implicit regression equations. What should be minimized is the squared difference between the dependent variable X_t and the prediction of X_t. As a consequence, the least squares statistical theory cannot be applied to the e_t term in the Gemcat approach. Hypothesis testing is therefore difficult.

The second problem is more fundamental and also concerns Guastello's (1988) technique. The derivative of the cusp potential function is used as a basis for the computation of fit. The problem of the derivative is that it does not distinguish between maxima and minima of the potential function. The maxima, however, represent the inaccessible states (see Figure 1), and data points in this area should contribute greatly to the error in fit of the cusp model. Unfortunately, in Gemcat and Guastello's technique, this is not the case since both techniques do not distinguish between minima and maxima of the cusp potential function. This point is illustrated in Figure 2. As a

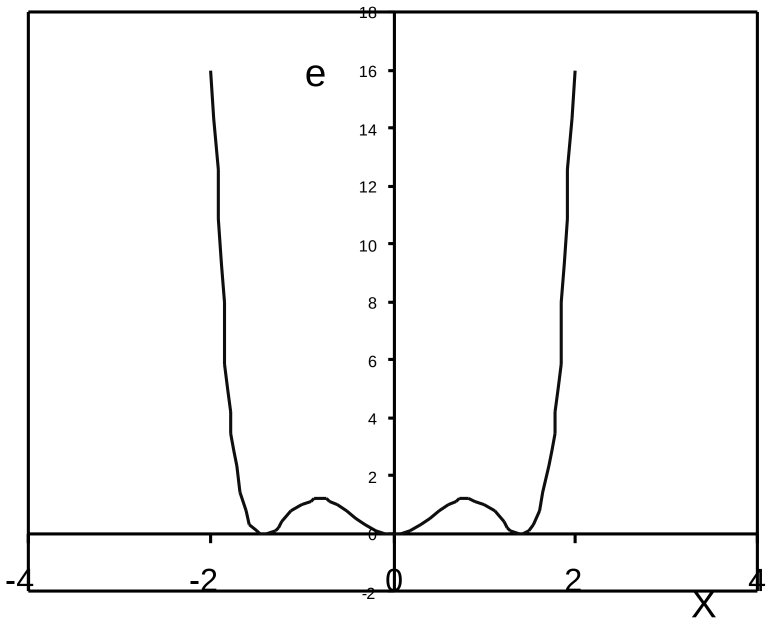

Figure 2: Error, e, as Function of X, According to the Gemcat Optimization Function (for α = 0 and β = 2)

NOTE: At $X = 0$, the potential function has a maximum and data points are not permitted in this so-called inaccessible state. However, the graph shows a minimum at $X = 0$, implying that data points around $X = 0$ do not contribute to the total error. Both in Gemcat and Guastello's (1988) technique, this occurs because they do not distinguish between minima and maxima of the cusp potential function.

consequence, the fit of the cusp model to any data set will be overestimated. It is difficult to solve this problem as long as the derivative of the potential function is used to fit data. The next technique applies the potential function of the cusp and does not suffer from this problem.

COBB'S METHOD

Cobb (1978, 1980, 1981) and Cobb and Zacks (1985) developed a catastrophe fitting technique that is based on a stochastic interpretation of catastrophe theory. The deterministic equation of motion, dX/dt, is extended with a stochastic noise term (i.e., a Wiener innovation). Assuming a constant variance of the innovation, the probability density function (PDF) for the cusp can be written as follows (for derivation, see Cobb and Zacks 1985; Hartelman 1997):

$$F(X|\alpha, \beta) = D \exp\left(-\frac{1}{4}X^4 + \frac{1}{2}\beta X^2 + \alpha X\right), \tag{1}$$

$$X = (Y - \lambda)/\sigma, \tag{2}$$

$$\alpha = a_0 + a_1 Z_1 + a_2 Z_2 + \ldots a_n Z_n, \tag{3}$$

$$\beta = b_0 + b_1 Z_1 + b_2 Z_2 + \ldots b_n Z_n, \tag{4}$$

where D is an integration constant depending on α and β. Parameters λ and σ scale the observed behavioral variable Y to X, and α and β are linear functions of the observed control variables Z_1 to Z_n. The behavioral variable is univariate, but the dependent variables may be multivariate. The parameters λ, σ, $a_0 \ldots a_n$ and $b_0 \ldots b_n$ are estimated using a maximum likelihood procedure (see Cobb and Watson 1980).

The inaccessible mode of the deterministic cusp model is reflected in the PDF by an area of low probability between the two high-probability modes (e.g., a bimodal distribution). Fitting the PDF of equation (1) to data does, in principle, not differ from fitting other probability densities such as the normal distribution. Conceptually, this procedure is much better than the other techniques since it is a relevant stochastic generalization of deterministic catastrophe theory. Yet, it has not been used often. A first reason might be that the derivation of the PDF is not easy to understand, but the most important reason is that Cobb's computer program often breaks down for nonapparent reasons.

This last problem was solved by Hartelman (1997) and Hartelman et al. (1998). Hartelman replaced the expectation-maximization routine that Cobb used by a quasi-Newton optimization routine, which is more reliable in this context. The gradient of the objective function is determined by a finite difference approximation. Hartelman also replaced the numerical integrator of Cobb by the numerical integrator D01AMF of the NAG library. These computational improvements lead to a much better performance of Cobb's technique, although sometimes sensitivity to starting values remains a problem (as with maximum likelihood estimation of many other distributions).

The starting value problem may be solved by trying several sets of starting values. All solutions reported in this article are maxima of the likelihood function obtained by varying starting values. Hartelman's (1997) program gives unstandardized and standardized (given standardization of observed variables) estimates of the parameters. These parameters or weights of the observed independent variables on the normal and splitting factors indicate how strongly the control variables are associated with the normal and splitting factor (as well as the independence between the control variables). Furthermore, the data points can be placed in the control plane, spanned by α and β. This shows in which part of the cusp data points are located.

The fit of the cusp model is compared with the fit of a multivariate logistic and a linear regression model. The logistic model allows for rapid change of the dependent variable but does not incorporate real jumps. Within the class of noncatastrophe models, it is the closest to the catastrophe models. The logistic model is defined as $Y = \lambda + \sigma/(1 + \exp(\alpha))$, where $\alpha = a_0 + a_1 Z_1 + a_2 Z_2 + \ldots a_n Z_n$. A direct comparison of log-likelihoods is not feasible since these models are not nested. The stochastic cusp model uses more parameters ($\#P$), and comparison should control for this. As measures of goodness of fit, Hartelman's program uses the Akaike information criterion (AIC) and Bayes information criterion (BIC). The AIC (Akaike 1974) penalizes the log-likelihood (LL) for the number of parameters (–2 LL + $\#P * 2$). In the BIC (Schwarz 1978), the penalty is a function of the number of subjects (N) and the number of parameters (–2 LL + $\#P * \mathrm{LN}(N)$). The model with the lowest AIC and BIC is selected. In case the AIC and BIC contradict each other, differences between models are usually too small to make clear-cut decisions. If models are nested (e.g., restricted vs. unrestricted cusp models), they can be compared by the likelihood ratio. Under certain mild regularity conditions, $-2(\ln L_{\text{unrestricted}} - \ln L_{\text{restricted}})$ is known to be chi-square distributed with the difference in the number of estimated parameters as degrees of freedom.

Hartelman (1997) reports simulations demonstrating accurate estimation of the parameters for sample sizes larger than 200. The distribution of the estimates is also reasonably normal. Simulations also show that the AIC and BIC favor the simpler logistic model when data are fitted that are generated with the cusp model but lie outside the bifurcation set (i.e., the area that gives rise to the characteristic flags).

The last important extension of Cobb's technique in the program of Hartelman (1997) is the possibility of introducing restrictions to parameters to test specific hypotheses (by fixing some of the parameters λ, σ, $a_0 \ldots a_n$ and $b_0 \ldots b_n$ to certain values, e.g., 0). In the most strict cusp catastrophe model for attitudes, the informational variable only loads on the normal factor, while involvement only loads on the splitting factor. In this case, the two variables have an independent joint effect on the attitudinal variable. This model can be tested by restricting some of the $a_0 \ldots a_n$ and $b_0 \ldots b_n$ parameters to 0 and can be compared (by AIC and BIC) with the unrestricted cusp model. In the next section, we present two examples to illustrate the usefulness of Hartelman's (1997) extension of Cobb's technique (for another recent application involving parameter restrictions, see Ploeger, van der Maas, and Hartelman 2002).

EXAMPLES

In the first example we reanalyze the data used by Latané and Nowak (1994) to demonstrate that higher levels of involvement, as measured by intensity of feeling (Study 1) and judged importance (Studies 2 and 3), increase the modality of the behavior or attitude toward an issue (Study 1) and favorableness toward political statements (Studies 2 and 3). These data sets do not include measures of normal variables. There is only one independent control variable that should load only on the splitting factor of the cusp (the loading on the normal factor is fixed to 0). Hence, the cusp model can only be partly tested with these data. This test (focusing on divergence) is in itself very informative since a good fit of the model strongly supports the claims of Latané and Nowak.

In the second example, we analyze data of a study on social cultural developments in the Netherlands (Felling, Peters, and Schreuder 1985). These data include attitudinal (dependent) variables, variables that can be interpreted as involvement (splitting) factors, and a variable that might operate as an information (normal) factor. This last factor is political orientation. The idea is that a position on this dimension reflects the kind of information one has at one's disposal about politics. This assumption is made because it is not possible to assess the information directly from the respondents. Even though the normal factor does not involve a direct measure of information, the complete cusp model can still be fitted on these data.

EXAMPLE 1: THE LATANÉ AND NOWAK STUDIES (USING ONE CONTROL VARIABLE)

To test the cusp model of attitudes, Latané and Nowak (1994) report three studies. They show that variance in attitude increases as involvement increases (Study 1) and that perceived importance is related to attitude extremity (Studies 2 and 3). The increased variance is explained by the emerging bimodality of the distribution of the responses. By fitting a constrained cusp model, this hypothesis can be tested more directly. Following the description of the three original studies, the results of the cusp fits are presented. Within each study, two model fits are performed: one with and one without restrictions. In the restricted model, involvement loads only on the splitting factor.

In Study 1, Latané and Nowak (1994) used a data set from Stouffer et al. (1950). In this study, U.S. soldiers were asked their opinion about three issues: the point system for demobilization, post-war conscription, and the Women's Army Corps. Attitudes were scaled by counting the number of questions on each issue to which a soldier gave a favorable response and ranged from 0 (*unfavorable*) to 6 (*favorable*). In addition, respondents were asked to indicate how strongly they felt about their answer (from intensity 0 to intensity 5). A total number of 6,817 attitude scores were assessed. In Figure 3, the combined results of the attitudinal statements are shown in relation to intensity. At intensity 0, a normal distribution describes the data best: Most people have a neutral attitude toward the three statements. At intensity 5, however, a bimodal distribution of attitudinal values emerges: a small number of people have a neutral attitude, while most have a pro or contra attitudinal position. Variance increases regularly and significantly (Latané and Nowak 1994).

Because the program Cuspfit cannot handle more than 5,000 subjects, we took only 10% of the data simply by dividing the frequency of each score by 10 ($N = 685$).[2] The data set contained one control variable (intensity of feeling) and one dependent variable (the attitude).

In the second study (Latané and Nowak 1994), 100 undergraduate psychology students (62% females) were given 16 political statements to evaluate. Attitudes toward each statement ranged from –2 (*oppose*) to +2 (*favor*). Next they were asked to rate the importance of the statements (using a 5-point scale from 0 = *unimportant* to 4 = *important*). In addition, respondents had to predict the responses of their best friend, a male classmate, and a female classmate. Results of this study resemble those of Study 1: At importance level 0, a more or less normal distribution is observed because most people have a neutral attitude toward the statements. At importance level 4, visual inspection suggests the emergence of a bimodal distribution. The overall correlation between importance and extremity of attitudes is significant: $r = .42$, $p < .001$.

In their last study (Latané and Nowak 1994), 62 members of University of North Carolina fraternities and sororities (65% females) served as respondents. The main difference between this and the original study is the expansion of the attitude scale from 5 to 7 points (-3 = *oppose* to $+3$ = *favor*). Visual inspection of the results suggests the same pattern as both former studies: a shift from a normal to a bimodal distribution as importance increases.

Results of Example 1

The data sets of Latané and Nowak (1994) were reanalyzed with the program Cuspfit. According to the AIC and BIC, both the unrestricted and restricted cusp models fitted much better than the linear and logistic models. This finding was obtained for all three cases (see Table 1). In Study 1, the restricted cusp model did not show a worse fit than the unrestricted cusp model, $\chi^2(1) = -2(-876.8 - 876.9) = .2$, $p > .05$. Both the AIC and BIC are lower for the restricted model. In Studies 2 and 3, the unrestricted cusp model has a better fit than the restricted cusp, $\chi^2(1) = 52$, $p < .01$ and $\chi^2(1) = 34$, $p < .01$, respectively. Table 2 shows the standardized parameter estimates. In Study 1, the effect of intensity on the normal factor is negligible compared to its effect on the splitting factor ($a_1 = -.002$;

TABLE 1: Fit Statistics in the Latané and Nowak Studies: One Control Variable

	Log-Likelihood	*AIC*	*BIC*	*#P*
Study 1				
Linear	–972.0	1,950	1,964	3
Logistic	–971.9	1,952	1,970	4
Cusp	–876.8	1,766	1,793	6
Cusp restricted	–876.9	1,764	1,787	5
Study 2				
Linear	–2,374	4,754	4,770	3
Logistic	–2,141	4,291	4,312	4
Cusp	–1,691	3,395	3,427	6
Cusp restricted	–1,717	3,444	3,471	5
Study 3				
Linear	–1,451	2,907	2,922	3
Logistic	–1,359	2,726	2,745	4
Cusp	–1,052	2,116	2,146	6
Cusp restricted	–1,069	2,149	2,173	5

NOTE: AIC is the Akaike information criterion, BIC is the Bayes information criterion, and $\#P$ is number of estimated parameters. In Study 1, the restricted cusp model has the lowest AIC and BIC; in Studies 2 and 3, the unrestricted cusp model has the lowest AIC and BIC.

$b_1 = 1.002$). Hence, the restricted model ($a_1 = .0$) has lower AIC and BIC than the unrestricted model. In Studies 2 and 3, the effect of importance on the normal factor is larger but is still small compared to its effect on the splitting factor (Study 2: $a_1 = .205$, $b_1 = .999$; Study 3: $a_1 = .166$, $b_1 = .655$). Figure 3 gives a graphical display of the data and the fitted model of Study 1. Results clearly support the hypothesis of divergence of Latané and Nowak (1994).

EXAMPLE 2: USING TWO CONTROL VARIABLES

In this example, we used data from a study on social-cultural developments in the Netherlands (Felling et al. 1985). In this study, 3,003 respondents answered a questionnaire containing 800 attitude items about issues such as religion, value systems, work, politics, ecology, health, and ethnocentrism. We focused on politics because a well-defined normal factor could be extracted. This variable was operationalized by the request to indicate a political position on a 10-point scale (1 = *left wing* and 10 = *right wing*). The second control

TABLE 2: Standardized Parameter Estimates of the Restricted and Unrestricted Cusp Models of Studies 1, 2, and 3

	Normal Factor (a)	*Splitting Factor (b)*	*Location (λ)*	*Scale (σ)*
Study 1				
Unrestricted model				
0 Constant	–0.154	0.619	0.016	1.006
1 Intensity	–0.002	1.002		
Restricted model				
0 Constant	–0.017	0.619	0.017	1.006
1 Intensity	0.000[a]	1.002		
Study 2				
Unrestricted model				
0 Constant	0.365	1.777	–0.539	0.852
1 Importance	0.205	0.999		
Restricted model				
0 Constant	0.451	1.730	–0.589	0.870
1 Importance	0.000[a]	0.115		
Study 3				
Unrestricted model				
0 Constant	0.275	2.128	–0.417	0.783
1 Importance	0.166	0.655		
Restricted model				
0 Constant	0.319	2.119	–0.440	0.789
1 Importance	0.000[a]	0.749		

a. Fixed

variable (involvement) was represented in this study by 12 items. A principal component analysis was carried out on the 12 involvement items. The following involvement measures load on the same factor: “interest in politics,” “no idea about political future,” “read about politics in newspapers,” “important to engage in politics,” “inquiries about politics,” “ideas about political future,” “little knowledge about politics,” and “discuss politics with other people.” The factor scores of this factor are used.

We selected 24 different attitudinal statements as possible behavioral variables. The statements are selected on the basis of the correlation with the political orientation variable, as well as the increase

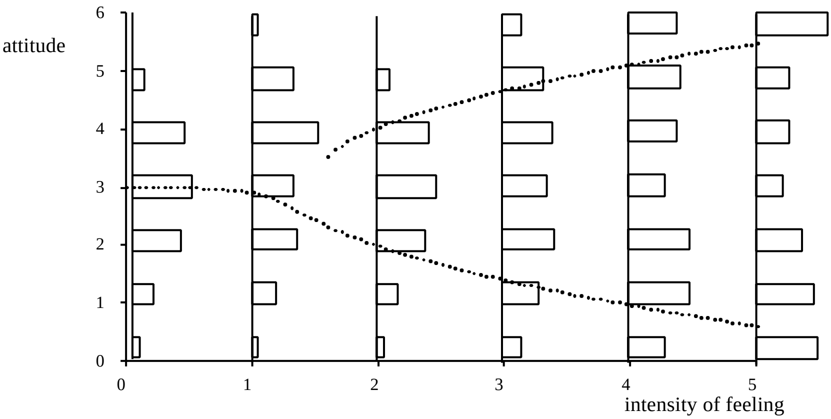

Figure 3: Fit of the Cusp Model to Data of Latané and Nowak

NOTE: For five intensities of feeling, the frequencies of the attitude scores are displayed. The dotted line displays the predicted values of the cusp model, indicating an increase of bimodality with higher values of intensity of feeling.

of bimodality of the distribution of the responses when involvement was increased. The fit of the cusp catastrophe model was better than that of the linear and logistic models in all 24 cases. However, not all analyses showed a clear result: In most cases, involvement and political orientation were not completely independent. The left-wing position was associated with more involvement. In the 24 analyses, the correlation between political orientation and involvement varied between .017 and .101 (in the latter case, $p < .05$), with left-wing respondents being more involved. In 8 cases, involvement could be interpreted as the sole splitting variable, and political orientation could be interpreted as the sole normal variable. Below, we report analysis of a typical (imperfect) case in more detail. The chosen attitudinal statement is as follows: “The government must force companies to let their workers benefit from the profit as much as the shareholders do” (1 = *totally agree*, 5 = *totally disagree*). For illustrative purposes, all 16 possible cusp models are fitted—for example, all 16 possibilities of restricting the loadings of the two control factors (4^2, one model without restrictions included).

TABLE 3: Fit Statistics of Example 2

Model	*Log-Likelihood*	*AIC*	*BIC*	*#P*	*Parameters Fixed at 0*
Linear	–2,065	4,139	4,160	4	
Logistic	–1,868	3,746	3,773	5	
Cusp 1	–1,908	3,823	3,844	4	a1, a2, b1, b2
Cusp 2	–1,901	3,811	3,838	5	a1, a2, b1
Cusp 3	–1,906	3,822	3,848	5	a1, b1, b2
Cusp 4	–1,900	3,812	3,843	6	a1, b1
Cusp 5	–1,873	3,755	3,781	5	a1, a2, b2
Cusp 6	–1,866	3,743	3,774	6	a1, a2
Cusp 7	–1,868	3,749	3,780	6	a1, b2
Cusp 8	–1,864	3,741	3,778	7	a1
Cusp 9	–1,816	3,642	3,668	5	a2, b1, b2
Cusp 10	–1,809	3,631	3,662	6	a2, b1
Cusp 11	–1,811	3,634	3,666	6	b1, b2
Cusp 12	–1,805	3,625[a]	3,661[a]	7	b1
Cusp 13	–1,816	3,644	3,675	6	a2, b2
Cusp 14	–1,809	3,631	3,668	7	a2
Cusp 15	–1,811	3,636	3,672	7	b2
Cusp 16	–1,805	3,626	3,668	8	

NOTE: AIC is the Akaike information criterion, BIC is the Bayes information criterion, and #P is number of estimated parameters.
a. The lowest value for AIC and BIC.

Results of Example 2

Table 3 shows the goodness-of-fit statistics of the 16 cusp models. The model that shows the best fit is Model 12 (according to both the likelihood ratio tests and the AIC/BIC). This model has also a lower AIC and BIC than either the linear or the logistic model. Although Model 12 has the lowest AIC and BIC, Model 10 has almost the same BIC as Model 12. Model 10 is the most stringent model to test for the cusp catastrophe hypothesis (political orientation only loads on the normal factor, and involvement only loads on the splitting factor). In comparison, Model 12 is marginally less strict (political orientation only loads on the normal factor). Table 4 shows the standardized parameter estimates of Models 10, 12, and 16. Involvement had a modest impact on the normal factor (Model 12: $a_2 = -.097$; Model 16: $a_2 = -.093$), while political orientation had a modest effect on the splitting factor (Model 16: $b_1 = .079$). These effects are much smaller, though, than the estimated effects of involvement on the

TABLE 4: Standardized Parameter Estimates of Restricted Models 12, 10, and 16 (Example 2)

	Normal (a)	*Splitting (b)*	*Location (l)*	*Scale (s)*
Model 10				
0 Constant	0.178	0.120	–0.134	1.143
1 Political Orientation	0.450	0.000[a]		
2 Involvement	0.000[a]	–0.250		
Model 12				
0 Constant	0.152	0.120	–0.120	1.141
1 Political Orientation	0.463	0.000[a]		
2 Involvement	–0.097	–0.230		
Model 16				
0 Constant	0.161	0.128	–0.137	1.140
1 Political Orientation	0.457	0.079		
2 Involvement	–0.093	–0.239		

splitting factor (Models 10, 12, and 16, respectively: $b_2 = -.250$, $-.230$, and $-.239$) and political orientation on the normal factor (Models 10, 12, and 16, respectively: $a_1 = .450, .463, .457$).

Figure 4 displays the fit of Model 12. The data are plotted in the control plane (see also Figure 1). It shows that political orientation only loads on the normal axis and that involvement mainly loads on the splitting axis. A reasonable portion of the data falls within the bifurcation set.

DISCUSSION

Catastrophe theory provides a way to model and test transitions in attitudes. Hypotheses can be formulated and tested by making use of the catastrophe flags. In doing so, different predictions can be brought together into one coherent model. Examples of issues that can be explained by the cusp catastrophe model are polarization and ambivalence. In our view, existing models of attitude (and stereotype) change, such as the bookkeeping, conversion, and subtyping models, can also be incorporated in the cusp model. In addition, this model includes a less well-studied mechanism of attitude change: the sudden

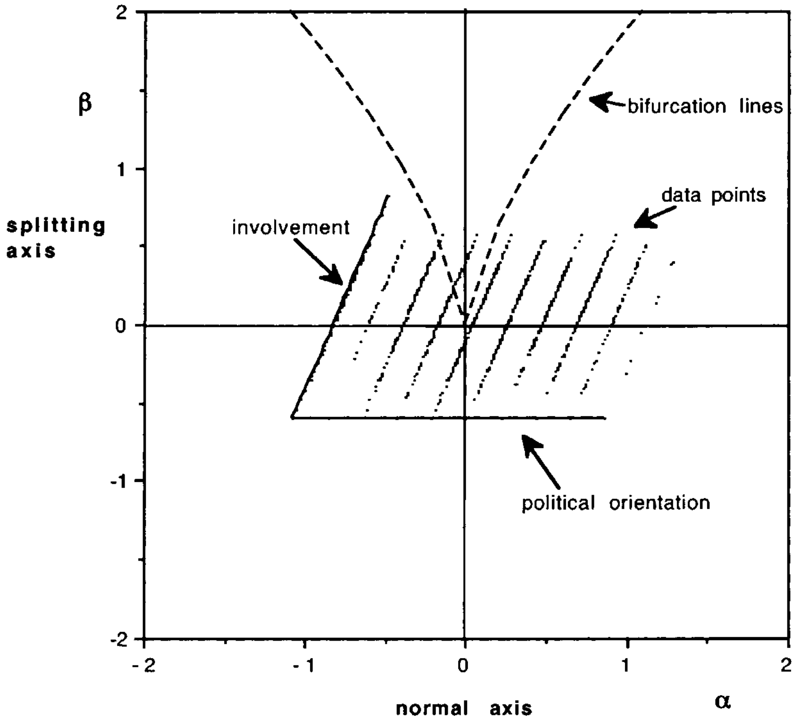

Figure 4: Location of the Data from Example 2 in the Cusp Control Plane

NOTE: The figure shows involvement and political orientation in relation to the splitting and normal factor.

change of an attitude or stereotype after a gradual increase of (one-sided) information.

We emphasized that the often-heard criticism that catastrophe models lack rigorous statistical tests can now be discarded. The way we tested the cusp model of attitude change meets the standards used in most fields of mathematical and statistical modeling in psychology. In our two examples (distinguished by making use of one or two control variables), the data are directly fitted in the cusp catastrophe model. In both examples, the cusp model fits the data better than either the linear or the logistic model.

In the first example, the results from the Stouffer et al. (1950) data set clearly indicated that involvement can be understood as the splitting factor. In the two other data sets, involvement contributed relatively little to the normal factor. In the second example, involvement and a left-wing political orientation were correlated. For this reason, the involvement measures could not exclusively be taken as splitting variables since they also measured some informational content. In future research, a distinction should be made between different types of involvement. These types of involvement activate different types of informational factors (Johnson and Eagly 1990). The results of the second example suggest that involvement is a splitting factor in relation to political attitudes, while political orientation works as the main informational factor. Better results are expected when direct measures of information and involvement are used.

A possible way to solve the measurement problem is suggested in van der Pligt et al. (2000), who describe attitudes as an associative network of attributes, representing positive and negative aspects of the attitude. How many and which attributes are part of the network can be determined by questionnaires or protocol analysis. van der Pligt et al. quantify ambivalence (A) by the following formula: $A = (Np + Nn)/2 - |Np - Nn|$, where Np and Nn are the number of positive and negative attributes. The formula states that ambivalence is a function of the sum of attributes minus the absolute difference between the number of positive and negative attributes (weighing of the importance of attributes might be necessary).

Based on this idea, we suggest defining involvement as the sum of the number of positive and negative attributes ($\beta = Np + Nn$) and information as the difference between the number of positive and negative attributes ($\alpha = Np - Nn$). As argued earlier, in the cusp model, ambivalence occurs when β is high and α is close to zero, which corresponds nicely with the formula of van der Pligt et al. (2000). Polarization (divergence) occurs when α is about zero and β strongly increases (e.g., when the number of positive and negative attributes increases equally without distorting the balance between them). Hysteresis can also be defined in these terms. This more refined definition of information and involvement might help to solve the problem with the operationalization of the control variables that was the major problem in our analyses.

As argued in this article, the model based on catastrophe theory can now be tested statistically. This development, in combination with a more refined measurement of the various factors, offers new opportunities for our understanding of attitudes and attitude change.

NOTES

1. An anonymous reviewer suggested that the problem with Guastello's methods is mainly due to the reverse hierarchical entry method (and to problems relating to ergodicity and stationarity) and not to the difference equation per se. Dismissal of the reverse hierarchical entry method would indeed prevent the false conclusion in favor of the cusp for random data. Yet, we think that the use of the difference equation is also wrong because catastrophe theory deals with equilibrium behavior, whereas the equation of Guastello (1988) is about transient behavior. Furthermore, Guastello's method fails to distinguish between minima and maxima (see Figure 2).
2. A reanalysis with 50% of the data produced very similar results.

REFERENCES

Akaike, Hirotugo. 1974. "A New Look at the Statistical Model Identification." *IEEE Transactions on Automatic Control* 19:716-23.

Alexander, Ralph A., Glenn R. Herbert, Richard P. Deshon, and Paul J. Hanges. 1992. "An Examination of Least Squares Regression Modeling of Catastrophe Theory." *Psychological Bulletin* 111:366-74.

Beek, Peter J., Frans Verschoor, and Scott Kelso. 1997. "Requirements for the Emergence of a Dynamical Social Psychology." *Psychological Inquiry* 8:100-4.

Burlingame, Gary M. and Carolen A. Hope. 1997. "Dynamical Systems and Social Psychology: The Promise and the Pitfalls." *Psychological Inquiry* 8:104-10.

Castrigiano, Domencio P. L. and Sandra A. Hayes. 1993. *Catastrophe Theory*. Reading, MA: Addison-Wesley.

Cobb, Loren. 1978. "Stochastic Catastrophe Models and Multimodal Distributions." *Behavioral Science* 23:360-74.

———. 1980. "Estimation Theory for the Cusp Catastrophe Model." Pp. 772-76 in *Proceedings of the Section on Survey Research Methods*. Alexandria, VA: American Statistical Association.

———. 1981. "Parameter Estimation for the Cusp Catastrophe Model." *Behavioral Science* 26:75-78.

Cobb, Loren and Bill Watson. 1980. "Statistical Catastrophe Theory: An Overview." *Mathematical Modelling* 1:311-17.

Cobb, Loren and Shelemyahu Zacks. 1985. "Applications of Catastrophe Theory for Statistical Modeling in the Biosciences." *Journal of the American Statistical Association* 80:793-802.

Eagly, Alice H. and Shelly Chaiken. 1993. *The Psychology of Attitudes*. Fort Worth, TX: Harcourt Brace.

Eiser, J. Richard, ed. 1994. *Attitudes, Chaos & the Connectionist Mind*. New York: Blackwell.

Felling, Albert J. A., Jan Peters, and Osmund Schreuder. 1985. *Sociaal-culturele Ontwikkelingen in Nederland*. Vakgroep Methoden van Onderzoek, Katholieke Universiteit van Nijmegen, SWIDOC Steinmetz Archive Database, NL STAR DATA P1012.

Flay, Brian R. 1978. "Catastrophe Theory in Social Psychology: Some Applications to Attitudes and Social Behavior." *Behavioral Science* 23:335-50.

Gilmore, Robert. 1981. *Catastrophe Theory for Scientists and Engineers*. New York: John Wiley.

Guastello, Stephan J. 1988. "Catastrophe Modeling of the Accident Process: Organizational Subunit Size." *Psychological Bulletin* 103:246-55.

———. 1992. "Clash of the Paradigms: A Critique of an Examination of the Polynomial Regression Technique for Evaluating Catastrophe Theory Hypotheses." *Psychological Bulletin* 111:375-79.

Hartelman, Pascal A. I. 1997. *Stochastic Catastrophe Theory*. Dissertatie reeks 1997-2, Faculteit Psychologie, Universiteit van Amsterdam.

Hartelman, Pascal A. I., Han L. J. van der Maas, and Peter C. M. Molenaar. 1998. "Detecting and Modeling Developmental Transitions." *British Journal of Developmental Psychology* 16:97-122.

Johnson, Blair T. and Alice H. Eagly. 1990. "Involvement and Persuasion: Types, Traditions, and the Evidence." *Psychological Bulletin* 107:375-84.

Kaplan, Kalman J. 1972. "On the Ambivalence-Indifference Problem in Attitude Theory and Measurement: A Suggested Modification of the Semantic Differential Technique." *Psychological Bulletin* 77:361-72.

Krosnick, Jon A. and Richard E. Petty. 1995. "Attitude Strength: An Overview." Pp. 1-24 in *Attitude Strength: Antecedents and Consequences*, edited by Richard E. Petty and Jon A. Krosnick. Mahwah, NJ: Lawrence Erlbaum.

Kruglanski, Arie W., Russell W. Clement, and John T. Jost. 1997. "The New Wave of Dynamism: Will It Engulf the Field?" *Psychological Inquiry* 8:132-35.

Lange, Rense, Terence A. Oliva, and Sean R. McDade. 2000. "An Algorithm for Estimation Multivariate Catastrophe Models: GEMCAT II." *Studies in Nonlinear Dynamics and Econometrics* 4:137-68.

Latané, Bibb and Andrzej Nowak. 1994. "Attitudes as Catastrophes: From Dimensions to Categories With Increasing Involvement." Pp. 219-249 in *Dynamical Systems in Social Psychology*, edited by R. R. Vallacher and A. Nowak. San Diego: Academic Press.

Latané, Bibb, Andrzej Nowak, and James H. Liu. 1994. "Measuring Emergent Social Phenomena: Dynamism, Polarization, and Clustering as Order Parameters of Social Systems." *Behavioral Science* 39:1-24.

McCutcheon, Allen L. 1987. *Latent Class Analysis*. Beverly Hills, CA: Sage.

McGarty, Craig and S. Alexander Haslam. 1997. *The Message of Social Psychology: Perspectives on Mind in Society*. Oxford, UK: Blackwell.

Oliva, Terence A., Wayne S. Desarbo, Diana L. Day, and Kamel Jedidi. 1987. "GEMCAT: A General Multivariate Methodology for Estimating Catastrophe Models." *Behavioral Science* 32:121-37.

Pettigrew, Thomas F. 1981. "Extending the Stereotype Concept." In *Cognitive Processes in Stereotyping and Intergroup Behavior*, edited by D. L. Hamilton. Hillsdale, NJ: Lawrence Erlbaum.

Petty, Richard E., Duane T. Wegener, and Lenadre R. Fabrigar. 1997. "Attitudes and Attitude Change." *Annual Review of Psychology* 48:609-47.

Ploeger, Annemie, Han L. J. van der Maas, and Pascal Hartelman. 2002. “Catastrophe Analysis of Switches in the Perception of Apparent Motion.” *Psychonomic Bulletin & Review* 9:26-42.

Poston, Tim and Ian Stewart. 1978. *Catastrophe Theory and Its Applications*. London: Pitman.

Reardon, Richard and Abraham Tesser. 1982. *Is Conformity a Catastrophe?* Unpublished manuscript, University of Georgia, Athens.

Rothbart, Myron. 1981. “Memory Processes and Social Beliefs.” Pp. 145-82 in *Cognitive Processes in Stereotyping and Intergroup Behavior*, edited by D. L. Hamilton. Hillsdale, NJ: Lawrence Erlbaum.

Saris, Willem E. 1997. “Public Opinion About the EU Can Easily Be Swayed in Different Directions.” *Acta Politica* 32:406-35.

Schwarz, Gideon. 1978. “Estimating the Dimension of a Model.” *Annals of Statistics* 6:461-64.

Sherif, Carolyn W., Muzafer Sherif, and Roger E. Nebergall. 1965. *Attitude and Attitude Change: The Social Judgment-Involvement Approach*. Philadelphia: Saunders.

Stouffer, Samuel A., Louis Guttman, Edward A. Suchman, Paul F. Lazarsfeld, Shirley A. Star, and John A. Clausen. 1950. *Measurement and Prediction*. Princeton, NJ: Princeton University Press.

Sussman, Hector J. and Raphael S. Zahler. 1978. “Catastrophe Theory as Applied to the Social and Biological Sciences: A Critique.” *Synthese* 37:117-216.

Taylor, Shelley E. 1981. “A Categorization Approach to Stereotyping.” Pp. 83-113 in *Cognitive Processes in Stereotyping and Intergroup Behavior*, edited by D. L. Hamilton. Hillsdale, NJ: Lawrence Erlbaum.

Teahan, John E. 1975. “A Longitudinal Study of Attitude Shifts Among Black and White Police Officers.” *Journal of Social Issues* 31:47-56.

Tesser, Abraham. 1976. “Attitude Polarization as a Function of Thought and Reality Constraints.” *Journal of Research in Personality* 10:183-94.

———. 1980. “When Individual Dispositions and Social Pressure Conflict: A Catastrophe.” *Human Relations* 33:393-407.

Tesser, Abraham and John Achee. 1994. “Aggression, Love, Conformity, and Other Social Psychological Catastrophes.” Pp. 95-109 in *Dynamical Systems in Social Psychology*, edited by R. R. Vallacher and A. Nowak. San Diego: Academic Press.

Thom, René. 1972. *Stabilité Estructurelle et Morphogénesè*. New York: Benjamin.

Thompson, Megan M., Mark P. Zanna, and Dale W. Griffin. 1995. “Let’s Not Be Indifferent About (Attitudinal) Ambivalence.” Pp. 361-86 in *Attitude Strength: Antecedent and Consequence*, edited by R. E. Petty and J. A. Krosnick. Mahwah, NJ: Lawrence Erlbaum.

Titterington, D. Mike, Adrian F. M. Smith, and Udi E. Makov. 1985. *Statistical Analysis of Finite Mixture Distributions*. Chicester, UK: Wiley.

Tourangeau, Roger 1987. “Attitude Measurement: A Cognitive Perspective.” Pp. 149-62 in *Social Information Processing and Survey Methodology*, edited by H. Hippler, N. Schwarz, and S. Sudman. New York: Springer-Verlag.

Vallacher, Robin R., Andrzej Nowak, and J. Kaufman. 1994. “Intrinsic Dynamics of Social Judgment.” *Journal of Personality and Social Psychology* 67:20-34.

van der Maas, Han L. J. 1998. “The Dynamical and Statistical Properties of Cognitive Strategies: Relations Between Strategies, Attractors, and Latent Classes.” Pp. 161-76 in *Applications of Nonlinear Dynamics to Developmental Process Modeling*, edited by K. M. Newell and P. C. M. Molenaar. Mahwah, NJ: Lawrence Erlbaum.

van der Maas, Han L. J. and Peter C. M. Molenaar. 1992. “Stagewise Cognitive Development: an Application of Catastrophe Theory.” *Psychological Review* 99:395-417.

van der Maas, Han L. J., Maartje E. J. Raijmakers, Pascal A. I. Hartelman, and Peter C. M. Molenaar. 1999. "Reaction Time as Recovery Time After Pertubation." Pp. 159-69 in *Nonlinear Analysis of Developmental Processes*, edited by G. Savelsbergh, H. L. J. van der Maas, and P. van Geert. Amsterdam: KNAW.

van der Pligt, Joop, Nanne K. De Vries, Antony S. R. Manstead, and Frenk van Harreveld. 2000. "The Importance of Being Selective: Weighing the Role of Attribute Importance in Attitudinal Judgment." *Advances in Experimental Social Psychology* 32:135-200.

Weber, Renee and Jennifer Crocker. 1983. "Cognitive Processes in the Revision of Stereotypic Beliefs." *Journal of Personality and Social Psychology* 45:961-77.

Zimbardo, Philip G., Ebbe B. Ebbesen, and Cristina Maslach. 1977. *Influencing Attitudes and Changing Behavior: An Introduction to Method, Theory and Applications of Social Control and Personal Power*. 2d ed. Reading, MA: Addison-Wesley.

Zeeman, E. Cristopher. 1976. "Catastrophe Theory." *Scientific American* 234:65-83.

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